Guidance Notes EL 001
Basic Principles on Measurement Uncertainty

1. Evaluation of Uncertainty
   The uncertainty of the result of a measurement generally consists of several components. They can be grouped into two categories according to the method used to estimate their numerical values:
   - **Type A evaluation**
     Calculation of uncertainty is by statistical analysis through repetitive observations.
   - **Type B evaluation**
     Calculation of uncertainty is by means other than statistical analysis.

2. Modeling the Measurement Process
   - A measurand $Y$ can be determined from $N$ inputs quantities $X_1$, $X_2$, $X_3$ … $X_N$, through a function $f$:
     \[ Y = f(X_1, X_2, X_3 \ldots X_N) \]
   - An estimate of $Y$, denoted by $y$, is obtained from $x_1$, $x_2$, $x_3$ … $x_N$, the estimates of the input quantities $X_1$, $X_2$, $X_3$ … $X_N$, through the same function $f$:
     \[ y = f(x_1, x_2, x_3 \ldots x_N) \]
   - The uncertainty associated with the estimate $y$ is obtained by appropriately combining the estimated standard deviation (or standard uncertainty) of each of the input estimate $x_i$.

3. Type A Evaluation of Standard Uncertainty
   - The arithmetic mean for $n$ independent observations:
   - The standard deviation of the $n$ independent observations:
     \[ \bar{q} = \frac{1}{n} \sum_{k=1}^{n} q_k \]
     \[ s(q_k) = \sqrt{\frac{1}{n-1} \sum_{k=1}^{n} (q_k - \bar{q})^2} \]
   - The standard deviation of the mean (estimate the spread of the distribution of the means):
     \[ s(\bar{q}) = \frac{s(q_k)}{\sqrt{n}} \]
For an input estimate $x_i$ determined from $n$ repeated observations, the Type A standard uncertainty $u(x_i)$, with degrees of freedom $v$ is given by:

$$u(x_i) = s(\bar{q})$$

$$v_i = n - 1$$

Note: the degree of freedom should always be given when Type A evaluation of an uncertainty component is reported.

4. Type B Evaluation of Standard Uncertainty

Covert a quoted uncertainty to a standard uncertainty from the knowledge of the probability distribution of the uncertainty.

Commonly used probability distributions:

- Normal or Gaussian probability distribution
- Rectangular probability distribution

Degree of freedom is assumed to be infinite

![Normal Probability Distribution](image)

A normal distribution can be assumed when an uncertainty is quoted with a given confidence level. For example, a calibration report states that the uncertainty of a voltmeter is ± 0.1 V with a confidence level of 95%. The standard uncertainty of the voltmeter is given by:

$$u(x) = \sigma = \frac{k\sigma}{k} = 0.1 = 0.051 V$$

(Note: 95 % level of confidence has a coverage factor of 1.96)
When an uncertainty is given by maximum bound within which all values are equally probable, the rectangular distribution can be assumed. For example, the accuracy of a voltmeter of a specific range is quoted as ± 0.2 V. The standard uncertainty of the voltmeter is given by:

\[ u(x) = \frac{a}{\sqrt{3}} = \frac{0.2}{\sqrt{3}} = 0.115 \text{ V} \]

5. Combined Standard Uncertainty

The estimate of a measurand \( Y \) is given by:

\[ Y = f(x_1, x_2, x_3, \ldots, x_N) \]

\[ \Delta y = \frac{\partial f}{\partial x_1} \Delta x_1 + \frac{\partial f}{\partial x_2} \Delta x_2 + \frac{\partial f}{\partial x_3} \Delta x_3 + \Lambda + \frac{\partial f}{\partial x_N} \Delta x_N \]

It can be shown that the above equation leads to:

\[ u_c^2 (y) = \left( \frac{\partial f}{\partial x_1} \right)^2 u^2 (x_1) + \left( \frac{\partial f}{\partial x_2} \right)^2 u^2 (x_2) + \left( \frac{\partial f}{\partial x_3} \right)^2 u^2 (x_3) + \Lambda + \left( \frac{\partial f}{\partial x_N} \right)^2 u^2 (x_N) \]

\[ = c_1^2 u^2 (x_1) + c_2^2 u^2 (x_2) + c_3^2 u^2 (x_3) + \Lambda + c_N^2 u^2 (x_N) \]

The combined standard uncertainty:

\[ u_c (y) = \sqrt{c_1^2 u^2 (x_1) + c_2^2 u^2 (x_2) + c_3^2 u^2 (x_3) + \Lambda + c_N^2 u^2 (x_N)} \]

where \( c_1, c_2, c_3, \ldots, c_N \) are the sensitivity coefficients
Each component of the combined standard uncertainty could be calculated using either Type A or Type B evaluation method.

6. Coverage Factor of Combined Uncertainty

To determine the coverage factor of combined uncertainty, the effective degree of freedom must be first calculated from the Welch-Satterthwaite formula:

$$v_{\text{eff}} = \frac{u^4_c(y)}{\sum_{i=1}^{N} c_i^4 \frac{u^4(x_i)}{v_i}}$$

Based on the calculated $v_{\text{eff}}$, obtain the $t$-factor $t_p(v_{\text{eff}})$ for the required level of confidence $p$ from the $t$-distribution table.

The coverage factor will be:

$$k_p = t_p(v_{\text{eff}})$$

7. Expanded Uncertainty

The expanded uncertainty defines an interval about the estimated result $y$ within which the true value of the measurand $Y$ is confidently believed to lie. It is given by:

$$U = k_p \cdot u_c(y)$$

The measurand $Y$ is reported in the following format:

$$Y = y \pm U$$

It means that the true value of measurand $Y$ is confidently believed to fall within the following range:

$$y - U \leq Y \leq y + U$$
Example #1: Resistance Measurement

A milliohm meter is used to measure the resistance of a current shunt resistor. At the selected range of the meter for the measurement, the calibration certificate states an uncertainty of $\pm 0.2$ m$\Omega$ at 95% of confidence level. Effects of room temperature and humidity on the measurement are found to be negligible.

![Milliohm Meter and Current Shunt Resistor](image)

Measurement record:

<table>
<thead>
<tr>
<th>Reading</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$ (m$\Omega$)</td>
<td>9.4</td>
<td>9.1</td>
<td>9.4</td>
<td>9.8</td>
<td>9.7</td>
<td>9.4</td>
<td>9.8</td>
<td>9.7</td>
<td>9.4</td>
<td>9.4</td>
</tr>
</tbody>
</table>

1. **Measurement Process Model**

   The measured resistance is given by:

   $$R_x = R_{rdg} + \Delta R_m$$

   where $R_{rdg}$: resistance reading recorded by the meter
   $\Delta R_m$: meter uncertainty

2. **Uncertainty Equation**

   The combined standard uncertainty is given by:

   $$u_c(R) = \sqrt{c_1^2 u^2(R_{rdg}) + c_2^2 u^2(\Delta R_m)}$$

   Since $c_1 = \frac{\partial R}{\partial R_{rdg}} = 1$ and $c_2 = \frac{\partial R}{\partial (\Delta R_m)} = 1$, the combined standard uncertainty is given by:

   $$u_c(R) = \sqrt{u^2(R_{rdg}) + u^2(\Delta R_m)}$$

   where
\( u(R_{rdg}) \) is the standard uncertainty due to the repeatability of the meter reading
\( u(\Delta R_m) \) is the standard uncertainty due to the meter calibration

### 3. Calculation of Uncertainty Components

**Type A evaluation:**
The best estimate of the measured resistance is given by the arithmetic mean:

\[
\overline{R} = \frac{1}{10} \sum_{i=1}^{10} R_i = \frac{1}{10}(95.1) = 9.51 \text{ m}\Omega
\]

Standard deviation:

\[
s(R) = \sqrt{\frac{1}{10-1} \sum_{i=1}^{10} (R_i - \overline{R})^2} = \frac{1}{\sqrt{9}}(2.449) = 0.522 \text{ m}\Omega
\]

Standard uncertainty:

\[
u(R_{rdg}) = s(\overline{R}) = \frac{s(R)}{\sqrt{n}} = \frac{0.522}{\sqrt{10}} = 0.165 \text{ m}\Omega
\]

Degree of freedom, \( \nu = 9 \)

**Type B evaluation:**
The uncertainty of the calibration is \( \pm 0.2 \text{ m}\Omega \) with 95 % of confidence level (\( k = 1.96 \)).

\[
u(\Delta R_m) = \frac{0.2}{1.96} = 0.102 \text{ m}\Omega
\]

Degree of freedom, \( \nu = \infty \)

Note: The value of 0.2 mΩ is used as a component for Type B evaluation on the assumption that the drift and stability of the equipment is negligible.

### 4. Uncertainty Budget Table

<table>
<thead>
<tr>
<th>Source of Uncertainty</th>
<th>Type</th>
<th>Uncertainty Value (mΩ)</th>
<th>Probability Distribution</th>
<th>( k )</th>
<th>( u_i \text{(mΩ)} )</th>
<th>( c_i )</th>
<th>( c_i \times u_i )</th>
<th>( \nu_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Repeatability ( u(R_{rdg}) )</td>
<td>A</td>
<td>0.165</td>
<td>-</td>
<td>-</td>
<td>0.165</td>
<td>1</td>
<td>0.165</td>
<td>9</td>
</tr>
<tr>
<td>Meter Calibration ( u(\Delta R_m) )</td>
<td>B</td>
<td>0.200</td>
<td>Normal</td>
<td>1.96</td>
<td>0.102</td>
<td>1</td>
<td>0.102</td>
<td>( \infty )</td>
</tr>
</tbody>
</table>

### 5. Combined Standard Uncertainty
\[ u_c(R) = \sqrt{0.165^2 + 0.102^2} = 0.194 \, \text{m}\Omega \]

6. **Effective Degrees of Freedom**

\[ v_{\text{eff}} = \frac{0.194^4}{0.165^4} + \frac{0.102^4}{\infty} \approx 17 \]

7. **Expanded Uncertainty**

For \( v_{\text{eff}} = 17 \), the coverage factor of the combined standard uncertainty \((k_p)\) is equal to 2.11 at 95 % level of confidence.

\[ U = k_p \times u_c = 2.11 \times 0.194 = 0.409 \, \text{m}\Omega \]

8. **Reporting of Result**

\[ R = 9.51 \pm 0.409 \, \text{m}\Omega \]

The measured resistance of the current shunt resistor is 9.51 m\(\Omega\). The expanded uncertainty is ± 0.409 m\(\Omega\) with a coverage factor of 2.11, assuming a normal distribution at a level of confidence of 95 %. 
Example #2: Temperature Measurement

A digital thermometer with a Type K thermocouple is used to measure the temperature inside a temperature chamber. The temperature controller of the chamber is set at 400°C.

![Diagram of temperature measurement setup]

Digital thermometer specification:

- Accuracy = ± 0.6 °C

Thermocouple specifications:

- Temperature correction for the thermocouple at 400 °C is 0.5 ± 1.0 °C at 95% confidence level
- Deviation due to immersion = ± 0.1 °C
- Deviation due to drift = ± 0.2 °C

Measurement record:

<table>
<thead>
<tr>
<th>S/N</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>T (°C)</td>
<td>400.1</td>
<td>400.0</td>
<td>400.1</td>
<td>399.9</td>
<td>399.9</td>
<td>400.0</td>
<td>400.1</td>
<td>400.2</td>
<td>400.0</td>
<td>399.9</td>
</tr>
</tbody>
</table>

1. Measurement Process Model

The measured temperature is given by:

\[ t_s = t_{rdg} + \Delta t_m + \Delta t_{tc} + \Delta t_{imm} + \Delta t_{drift} \]

where

- \( t_{rdg} \) is the temperature reading recorded by the digital thermometer
- \( \Delta t_m \) is the accuracy of the digital thermometer
- \( \Delta t_{tc} \) is the temperature correction of the thermocouple

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$\Delta t_{imm}$ is the deviation due to immersion of the thermocouple

$\Delta t_{drift}$ is the deviation due to drift of the thermocouple

2. Uncertainty Equation

$$u_c(t_x) = \sqrt{u^2_{tdg} + u^2_{dm} + u^2_{tc} + u^2_{imm} + u^2_{drift}}$$

All the sensitivity coefficients are equal to unity.

3. Calculation of Uncertainty Components

Type A evaluation:
The best estimate of the measured temperature is given by the arithmetic mean:

$$\bar{T} = \frac{1}{10} \sum_{k=1}^{10} T_k = 400.02 \ ^\circ C$$

Standard deviation:

$$s(T) = \frac{1}{\sqrt{10 - 1}} \sqrt{\sum_{k=1}^{10} (T_k - \bar{T})^2} = 0.103 \ ^\circ C$$

Standard uncertainty:

$$u(t_{deg}) = \frac{s(T)}{\sqrt{n}} = \frac{0.103}{\sqrt{10}} = 0.033 \ ^\circ C$$

Degree of freedom, $\nu = 9$

Type B evaluation:
The accuracy of the digital thermometer = $\pm 0.6 \ ^\circ C$. Assume rectangular distribution, the standard uncertainty of the digital thermometer meter:

$$u(\Delta t_{dev}) = \frac{0.6}{\sqrt{3}} = 0.346 \ ^\circ C$$

Degree of freedom, $\nu = \infty$

The uncertainty of the temperature correction of the thermocouple = $\pm 1.0 \ ^\circ C$ at 95% confidence level ($k = 1.96$). The standard uncertainty due to temperature correction:

$$u(\Delta t_{tc}) = \frac{1.0}{1.96} = 0.510 \ ^\circ C$$

Degree of freedom, $\nu = \infty$
The uncertainty of the thermocouple due to immersion = ± 0.1 °C. Assume rectangular distribution, the standard uncertainty due to immersion:

$$u(\Delta_{imm}) = \frac{0.1}{\sqrt{3}} = 0.058 \, ^\circ\text{C}$$

Degree of freedom, $\nu = \infty$

The uncertainty of the thermocouple due to drift = ± 0.2 °C. Assume rectangular distribution, the standard uncertainty due to drift:

$$u(\Delta_{drift}) = \frac{0.2}{\sqrt{3}} = 0.115 \, ^\circ\text{C}$$

Degree of freedom, $\nu = \infty$

4. Uncertainty Budget Table

<table>
<thead>
<tr>
<th>Source of Uncertainty</th>
<th>Type</th>
<th>Uncertainty Value (°C)</th>
<th>Probability Distribution</th>
<th>$k$</th>
<th>$u_i$ (°C)</th>
<th>$c_i$</th>
<th>$c_i \times u_i$</th>
<th>$v_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Repeatability $u(t_{rdg})$</td>
<td>A</td>
<td>0.033</td>
<td>-</td>
<td>-</td>
<td>0.033</td>
<td>1</td>
<td>0.033</td>
<td>9</td>
</tr>
<tr>
<td>Digital Thermometer $u(\Delta_{tm})$</td>
<td>B</td>
<td>0.6</td>
<td>Rectangular</td>
<td>1.732</td>
<td>0.346</td>
<td>1</td>
<td>0.346</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Temperature correction $u(\Delta_{tc})$</td>
<td>B</td>
<td>1.0</td>
<td>Normal</td>
<td>1.96</td>
<td>0.510</td>
<td>1</td>
<td>0.510</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Immersion $u(\Delta_{imm})$</td>
<td>B</td>
<td>0.1</td>
<td>Rectangular</td>
<td>1.732</td>
<td>0.058</td>
<td>1</td>
<td>0.058</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Drift $u(\Delta_{drift})$</td>
<td>B</td>
<td>0.2</td>
<td>Rectangular</td>
<td>1.732</td>
<td>0.115</td>
<td>1</td>
<td>0.115</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

5. Combined Standard Uncertainty

$$u_{c}(t_c) = \sqrt{0.033^2 + 0.346^2 + 0.510^2 + 0.058^2 + 0.115^2} = 0.63 \, ^\circ\text{C}$$

6. Effective degrees of freedom

$$v_{eff} = \frac{0.63^4}{9 + \infty + \infty + \infty + \infty + \infty}$$

$$= 1,195,498$$

$$\approx \infty$$
7. Expanded Uncertainty

Degree of freedom for the combined standard uncertainty approaches $\infty$. Therefore, coverage factor of the combined standard uncertainty ($k_p$) is equal to 1.96 at 95 % level of confidence.

$$U = k_p \times u_c = 1.96 \times 0.63 = 1.235 \, ^\circ C$$

8. Reporting of result

The correction at 400 °C is 0.5 °C, hence

$$T = (400.02 + 0.5) \pm 1.235 \, ^\circ C = 400.52 \pm 1.235 \, ^\circ C$$

The measured temperature of the chamber is 400.52 °C. The expanded uncertainty is $\pm 1.235 \, ^\circ C$ with a coverage factor of 1.96, assuming a normal distribution at a level of confidence of 95 %.
Example #3: Current Measurement

A current of 10 A is measured by using a current shunt and a voltmeter.

Current shunt specifications:

- The calibration report gives \( R = 0.010088 \, \Omega \) at 10 A (23 °C) and expanded uncertainty = ± 0.08% at 95 % confidence level
- Temperature coefficient between 15 to 30 °C = 60 ppm/K
- Uncertainty due to resistance drift is negligible

Digital voltmeter specifications:

Under the condition of 15 to 40 °C

<table>
<thead>
<tr>
<th>Range</th>
<th>Full scale</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>200 mV</td>
<td>199.99 mV</td>
<td>±(0.03 % of reading + number of counts)</td>
</tr>
</tbody>
</table>

Measurement record:
Room temperature = 23 ± 5 °C

<table>
<thead>
<tr>
<th>Reading</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage (mV)</td>
<td>100.6</td>
<td>100.8</td>
<td>100.7</td>
<td>100.6</td>
<td>100.6</td>
<td>100.9</td>
<td>100.6</td>
<td>100.6</td>
<td>100.7</td>
<td>100.6</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>3</td>
<td>9</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>0</td>
<td>8</td>
<td>6</td>
<td>5</td>
</tr>
</tbody>
</table>

1. Measurement Process Model

\[ I = f(V, R) = \frac{V}{R} \]
2. Uncertainty Equation

\[
u_c^2(I) = \left( \frac{\partial I}{\partial V} \right)^2 u_1^2(V) + \left( \frac{\partial I}{\partial V} \right)^2 u_2^2(V) + \left( \frac{\partial I}{\partial R} \right)^2 u_3^2(R) + \left( \frac{\partial I}{\partial R} \right)^2 u_4^2(R)
\]

\[= c_1^2 [u_1^2(V) + u_2^2(V)] + c_2^2 [u_3^2(R) + u_4^2(R)]\]

The sensitivity coefficients:

\[c_1 = \frac{\partial I}{\partial V} = \frac{1}{R} \quad \text{and} \quad c_i = \frac{\partial I}{\partial R} = -\frac{V}{R^2}\]

where

\[u_1(V): \text{standard uncertainty of measured voltage due to repeatability}\]
\[u_2(V): \text{standard uncertainty of measured voltage due to voltmeter resolution}\]
\[u_3(R): \text{standard uncertainty of current shunt calibrated resistance value}\]
\[u_4(R): \text{standard uncertainty of current shunt resistance due to temperature effect}\]

3. Calculation of Uncertainty Components

Type A evaluation:

The best estimate of the measured voltage is given by the arithmetic mean:

\[\bar{V} = \frac{1}{10} \sum_{k=1}^{10} V_k = \frac{1}{10} (1007.2) = 100.72 \text{ mV}\]

Standard deviation:

\[s(V) = \sqrt{\frac{1}{10-1} \sum_{k=1}^{10} (V_k - \bar{V})^2} = \sqrt{\frac{1}{9} (1040 \times 10^{-4})} = 10.75 \times 10^{-2} \text{ mV}\]

Standard uncertainty:

\[u_1(V) = s(\bar{V}) = \frac{s(V)}{\sqrt{n}} = \frac{10.75 \times 10^{-2}}{\sqrt{10}} = 3.40 \times 10^{-2} \text{ mV}\]

Degree of freedom, \(\nu_f = 9\)

Type B evaluation:

The resolution of the voltmeter = ± 0.03 % of reading + 2 counts

\[= ± (0.03/100) \times 100.72 + 2(0.01) = ± 5.02 \times 10^{-2} \text{ mV}\]

Assuming rectangular distribution, the standard uncertainty due to voltmeter resolution:
\[ u_2(V) = \frac{5.02 \times 10^{-2}}{\sqrt{3}} = 2.90 \times 10^{-3} \text{ mV} \]

Degree of freedom, \( v_2 = \infty \)

The uncertainty of the shunt resistance is 0.08%.

Normal distribution with 95\% level of confidence \((k = 1.96)\)

\[ u_3(R) = \frac{8.07 \times 10^{-6}}{1.96} = 4.12 \times 10^{-6} \Omega \]

Degree of freedom, \( v_3 = \infty \)

The uncertainty of the shunt resistance due to temperature effect:

\[ 60 \times 10^{-6} \times \Delta t \times R = 60 \times 10^{-6} \times 5 \times 0.010088 = 3.03 \times 10^{-6} \Omega \]

Assuming rectangular distribution,

\[ u_4(R) = \frac{3.03 \times 10^{-6}}{\sqrt{3}} = 1.75 \times 10^{-6} \Omega \]

Degree of freedom, \( v_4 = \infty \)

\[ c_i = \frac{V}{R^2} = \frac{100.72 \times 10^{-3}}{0.010088^2} = -989.70 \text{ V}/\Omega^2 \]

4. Uncertainty Budget Table

<table>
<thead>
<tr>
<th>Source of Uncertainty</th>
<th>Type</th>
<th>Uncertainty Value</th>
<th>Probability Distribution</th>
<th>( k )</th>
<th>( u_i )</th>
<th>( c_i )</th>
<th>( c_i \times u_i ) (A)</th>
<th>( v_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltmeter Repeatability ( u_1(V) )</td>
<td>A</td>
<td>( 3.40 \times 10^2 ) mV</td>
<td>-</td>
<td>-</td>
<td>( 3.40 \times 10^{-2} ) mV</td>
<td>99.128 S</td>
<td>( 3.37 \times 10^{-3} )</td>
<td>9</td>
</tr>
<tr>
<td>Voltmeter Resolution ( u_2(V) )</td>
<td>B</td>
<td>( 5.02 \times 10^2 ) mV</td>
<td>Rectangular</td>
<td>1.732</td>
<td>( 2.90 \times 10^{-2} ) mV</td>
<td>99.128 S</td>
<td>2.87 \times 10^{-2}</td>
<td>( \infty )</td>
</tr>
<tr>
<td>Shunt Resistance ( u_3(R) )</td>
<td>B</td>
<td>( 8.07 \times 10^{-6} ) ( \Omega )</td>
<td>Normal</td>
<td>2</td>
<td>( 4.12 \times 10^{-6} ) ( \Omega )</td>
<td>989.7 ( \text{V}/\Omega^2 )</td>
<td>( 4.08 \times 10^{-3} )</td>
<td>( \infty )</td>
</tr>
<tr>
<td>Shunt Temp. Effect ( u_4(R) )</td>
<td>B</td>
<td>( 3.03 \times 10^{-6} ) ( \Omega )</td>
<td>Rectangular</td>
<td>1.732</td>
<td>( 1.75 \times 10^{-6} ) ( \Omega )</td>
<td>989.7 ( \text{V}/\Omega^2 )</td>
<td>( 1.73 \times 10^{-3} )</td>
<td>( \infty )</td>
</tr>
</tbody>
</table>

5. Combined Standard Uncertainty

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\[ u_c^2(I) = c_1^2 u_1^2(V) + c_2^2 u_2^2(V) + c_3^2 u_3^2(R) + c_4^2 u_4^2(R) = (3.37 \times 10^{-3})^2 + (2.87 \times 10^{-3})^2 + (4.08 \times 10^{-3})^2 + (1.73 \times 10^{-3})^2 \]

\[ u_c(I) = \sqrt{3.92 \times 10^{-5}} = 6.26 \times 10^{-3} \text{ A} \]

6. **Effective Degrees of Freedom**

\[ \nu_{eff} = \frac{(6.26 \times 10^{-3})^4}{(3.37 \times 10^{-3})^4} + \frac{(2.87 \times 10^{-3})^4}{\infty} + \frac{(4.08 \times 10^{-3})^4}{\infty} + \frac{(1.73 \times 10^{-3})^4}{\infty} \]

\[ = 107 \]

7. **Expanded Uncertainty**

Since \( \nu_{eff} = 10\& > 100 \), the coverage factor of the combined standard uncertainty (\( k_p \)) approaches 1.96 at 95 % level of confidence.

\[ U = k_p \times u_c = 1.96 \times 6.26 \times 10^{-3} = 0.012 \text{ A} \]

8. **Reporting of Results**

\[ \bar{I} = \frac{\bar{V}}{R} = \frac{100.72 \times 10^{-3}}{0.010088} = 9.984 \text{ A} \]

\[ I = 9.984 \pm 0.012 \text{ A} \]

The measured current is 9.984 A. The expanded uncertainty is ± 0.012 A with a coverage factor of 1.96, assuming a normal distribution at a level of confidence of 95 %.

**References:**

